

Bonn TK-96-06

## A covariant diquark-quark model of the nucleon in the Salpeter approach

V. Keiner

*Institut für Theoretische Kernphysik,  
Universität Bonn, Nussallee 14-16, D-53115 Bonn, FRG*

(March 5, 1996)

### Abstract

We develop a rather simple, formally covariant quark-diquark model of the nucleon. The nucleon is treated as a bound state of a constituent quark and a diquark interacting via a quark exchange. We include both scalar and axial-vector diquarks. The underlying Bethe-Salpeter equation is transformed into a pair of coupled Salpeter equations. The electromagnetic form factors of the nucleon are calculated in the Mandelstam formalism. We obtain a very good description of all electromagnetic form factors for momentum transfers up to  $-3 \text{ (GeV/c)}^2$ .

PACS numbers: 13.40.Gp, 13.39.Ki, 14.20.Dh, 11.10.St

e-mail: [keiner@pythia.itkp.uni-bonn.de](mailto:keiner@pythia.itkp.uni-bonn.de)

Tel.: +49 (0)228 73 2377

Fax: +49 (0)228 73 3728

## I. INTRODUCTION

In a recent paper we presented a simple, formally covariant scalar-diquark-quark model of the nucleon [1]. Combining the Salpeter approach with the Mandelstam formalism we calculated the electromagnetic form factors of the nucleon. Despite the simplicity of the model, we obtained a very good description of the measured proton electric form factor for momentum transfers up to  $-3 \text{ GeV}^2$ . However, the electric form factor of the neutron came out a factor of more than three too large, and the calculated magnetic form factors failed to describe the data quantitatively. Since scalar diquarks do not contribute to the magnetic current density, the magnetic form factors result only from the coupling to the quark. This is the main motivation for introducing also spin 1 diquarks. Of course, in the non-relativistic quark model, two quarks may couple to spin 0 or to spin 1, thus forming a mixed-antisymmetric or mixed-symmetric spin state. One main idea of the present model is to identify the mixed-antisymmetric part of the spin function with the scalar diquark channel and the mixed-symmetric one with the axial-vector diquark channel. An axial-vector particle has positive parity, and we consistently neglect any relative angular momentum between the diquark and the quark. There are indeed theoretical arguments in favour of a dominance of scalar diquarks [2,3], and also of eventual axial-vector diquarks [4,5]. Also, the amount of experimental hints of diquarks in the nucleon is not negligible, see [6] for a review on that subject. On the other hand, there are theoretical works which clearly deny such correlations, see e.g. [7]. However, we do not want to discuss against or in favour of diquark correlations. Obviously, this is only possible in a full three particle calculation. Rather, we develop a simple but formally covariant model of the nucleon which is able to reproduce various experimental data such as electromagnetic form factors, mean square charge radii and magnetic moments.

As in [1] we start from the Bethe-Salpeter equation. Following Salpeter [8] we assume an instantaneous interaction and obtain a Salpeter-type equation. As the only interaction we adopt an (instantaneous) quark exchange between the quark and the diquark. This interaction has been previously used in various studies [9–14,5] and seems most natural for a quark-diquark model. Involving only scalar and axial-vector diquark channels, we deduce a pair of coupled integral equations similar to [12]. Within a basis of positive parity amplitudes with spin  $\frac{1}{2}$ , we obtain a bound state solution below the threshold for this Schrödinger-type equation. Then, electromagnetic transition currents and form factors are calculated using the Mandelstam formalism [15]. For details of the calculation see [1].

This paper is structured as follows. In Sec. II we summarize the basic equations of the model. The pair of coupled Salpeter equations is presented. The hermiticity condition for the interaction is shortly outlined. Sec. III presents the idea of how the proton and neutron currents are calculated. The results of some current matrix elements and of the form factors are given in Sec. IV. We compare with experimental data. Finally, a summary is given in Sec. V.

## II. THE MODEL

### A. The diquark-quark Salpeter equation

A relativistic bound state of a scalar or (axial-)vector particle and a quark with total four momentum  $P$  is described by a Bethe-Salpeter amplitude

$$\chi_P(x_1, x_2)_{(\mu)\alpha} = \left\langle 0 \left| T \phi_{(\mu)}(x_1) \psi_\alpha(x_2) \right| P \right\rangle . \quad (1)$$

In momentum space,  $\chi_P$  fulfills the following Bethe-Salpeter equation [16]

$$\chi_P(p)_{(\mu)} = \Delta_1^F(p_1)_{(\mu)}^{(\nu)} S_2^F(p_2) \int \frac{d^4 p'}{(2\pi)^4} (-iK(P, p, p') \chi_P(p'))_{(\nu)} \quad (2)$$

with  $\Delta_1^F(p_1)_{(\mu)}^{(\nu)}$  and  $S_2^F(p_2)$  the Feynman propagators for a scalar (vector) particle and a quark, respectively, and  $-iK$  the irreducible interaction kernel. The fundamental equation of our model has been derived in [1]. It is a Schrödinger-type equation of the following form (all indices suppressed):

$$\begin{aligned} (\mathcal{H}\Psi)(\vec{p}) &= M \Psi(\vec{p}) \\ &= \frac{\omega_1 + \omega_2}{\omega_2} H_2(\vec{p}) \Psi(\vec{p}) + \frac{1}{2\omega_1} \int \frac{d^3 p'}{(2\pi)^3} W(\vec{p}, \vec{p}') \Psi(\vec{p}') \\ &= (\mathcal{T} + \mathcal{W}) \Psi(\vec{p}) . \end{aligned} \quad (3)$$

$\Psi(\vec{p})$  is the Bethe-Salpeter amplitude in the bound state's rest frame integrated over  $p^0$ :

$$\Psi(\vec{p}) = \gamma^0 \left( \int \frac{dp^0}{2\pi} \chi_P(p^0, \vec{p}) \right)_{P=(M, \vec{0})} . \quad (4)$$

$W(\vec{p}, \vec{p}') = K(P, p, p')\gamma^0$  is the interaction kernel in the rest frame which is supposed to be independent of  $p^0$  and  $p'^0$ .  $H_2(\vec{p})$  is the Dirac Hamiltonian of the quark, and  $\omega_i^2 = \vec{p}^2 + m_i^2$  the squared on-shell energies of the diquark and quark, respectively. The masses  $m_i$  are constituent masses of the order of a few hundred MeV, see Sec. IV.  $M$  is the mass of the bound state. For the normalization we obtained in [1]:

$$\langle \Psi | \Psi \rangle = 2M \quad (5)$$

with the scalar product defined as

$$\langle \Psi' | \Psi \rangle = \int \frac{d^3 p}{(2\pi)^3} (2\omega_1) \Psi'^{\dagger}(\vec{p}) \Psi(\vec{p}) . \quad (6)$$

To obtain real eigenvalues  $M$  of the pseudo-Hamiltonian  $\mathcal{H}$  in Eq. (3) the interaction kernel has to be chosen such that  $\mathcal{H}$  is hermitian with respect to the scalar product (6).

## B. The interaction kernel

Scalar and axial-vector diquarks (called v-diquarks in the following) couple to two quarks according to the Lagrangians [12]

$$\begin{aligned}\mathcal{L}_{int}^{scalar} &= -i g_s \bar{\psi}_c \gamma^5 \tau_2 \psi \phi^* , \\ \mathcal{L}_{int}^{axial-vector} &= -g_v \bar{\psi}_c \gamma^\mu \tau_a \psi \phi_\mu^{a*} ,\end{aligned}\quad (7)$$

respectively. Here,  $g_s$  and  $g_v$  are dimensionless coupling constants, and we will choose  $g_s = g_v = g$ , see Sec. IV. To obtain a stable solution of the Salpeter equation (3) (or (9), see below), we have to introduce a form factor of the diquarks, see [1]. It is assumed to be the same for the scalar and v-diquark and is chosen to be a Gaussian function. Again we use as the only interaction between the diquark and quark a quark exchange and perform the instantaneous approximation, i.e. neglect the  $p^0$  dependence. So the interaction kernel is of the form

$$W(\vec{p}, \vec{p}') = -g^2 \frac{1}{\omega_q^2} (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^0 , \quad (8)$$

with  $\omega_q = \sqrt{(\vec{p} + \vec{p}')^2 + m_q^2}$  the energy of the exchanged quark. We redefine the above  $g$  by absorbing the colour factor of 2 (For a given colour of the diquark there are two possible colours of the exchanged quark). We then obtain from Eq. (3) the following coupled integral equations for the scalar and v-diquark components of the nucleon, namely  $\Psi^{[0]}$  and  $\Psi_\mu^{[1]}$  (see Fig. 1) (without flavour dependence):

$$\begin{aligned}M\Psi^{[0]}(\vec{p}) &= \frac{\omega_1 + \omega_2}{\omega_2} H_2(\vec{p}) \Psi^{[0]}(\vec{p}) \\ &+ \frac{1}{2\omega_1} \int \frac{d^3 p'}{(2\pi)^3} (-g_s^2) \frac{1}{\omega_q^2} (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^0 \Psi^{[0]}(\vec{p}') \\ &+ \frac{1}{2\omega_1} \int \frac{d^3 p'}{(2\pi)^3} (-g_s g_v) \frac{1}{\omega_q^2} \gamma^\mu \gamma^5 (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^0 \Psi_\mu^{[1]}(\vec{p}') , \\ M\Psi_\mu^{[1]}(\vec{p}) &= \frac{\omega_1 + \omega_2}{\omega_2} H_2(\vec{p}) \Psi_\mu^{[1]}(\vec{p}) \\ &+ \frac{1}{2\omega_1} \int \frac{d^3 p'}{(2\pi)^3} (+g_v^2) \frac{1}{\omega_q^2} \gamma^\nu \gamma^5 (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^5 \gamma_\mu^\dagger \gamma^0 \Psi_\nu^{[1]}(\vec{p}') \\ &+ \frac{1}{2\omega_1} \int \frac{d^3 p'}{(2\pi)^3} (+g_s g_v) \frac{1}{\omega_q^2} (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^5 \gamma_\mu^\dagger \gamma^0 \Psi^{[0]}(\vec{p}') .\end{aligned}\quad (9)$$

The above interaction  $\mathcal{W}$  (compare with Eq. (3)) is indeed hermitian with respect to the scalar product (6), since the following conditions are fulfilled:

$$\begin{aligned}W^{[0][0]}(\vec{p}, \vec{p}')^\dagger &= W^{[0][0]}(\vec{p}', \vec{p}) \\ W^{[1][1]\mu}_\nu(\vec{p}, \vec{p}')^\dagger &= W^{[1][1]\nu}_\mu(\vec{p}', \vec{p}) \\ W^{[1][0]}_\mu(\vec{p}, \vec{p}')^\dagger &= W^{[0][1]\mu}(\vec{p}', \vec{p}) ,\end{aligned}\quad (10)$$

where e.g.

$$W^{[1][0]}_{\mu}(\vec{p}, \vec{p}') = (+g_s g_v) \frac{1}{\omega_q^2} (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^5 \gamma_{\mu}^{\dagger} \gamma^0. \quad (11)$$

The sign of  $W^{[0][0]}$  is chosen in [1] to obtain a bound state of only a scalar diquark and a quark. With respect to a further extension of our model (see Sec. V) we choose the sign of  $W^{[1][1]}$  to be positive. It can be shown that this sign gives rise to a bound  $\Delta(3/2)$  composed of a v-diquark and a quark.

### C. Solving the diquark-quark equation

The procedure solving Eq. (9) is analogous to [1] and involves the Ritz variational principle. As a basis of the nucleon with spin  $(\frac{1}{2}, s)$  in spin and Lorentz space we choose

$$e_s^{i[0]} = {}^t(e_s^i, 0, 0), \quad e_{s0}^{i[1]} = {}^t(0, e_s^i, 0), \quad e_{sV}^{i[1]} = {}^t(0, 0, e_s^i),$$

with  $e_s^1(\hat{p}) = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$ ,  $e_s^2(\hat{p}) = \begin{pmatrix} 0 \\ \frac{\vec{\sigma}\hat{p}}{W+M_{12}}\chi_s \end{pmatrix}$ , (12)

where  $M_{12} = \frac{m_1 m_2}{m_1 + m_2}$  and  $W^2 = \vec{p}^2 + M_{12}^2$ . Note that we combined the three vector components of the v-diquark to one reduced function only carrying total spin 1. The corresponding basis vector is denoted  $e_{sV}^{i[1]}$ . The contribution from the zero-component of the v-diquark is expanded in terms of  $e_{s0}^{i[1]}$  and that from the scalar diquark channel in terms of  $e_s^{i[0]}$ . However, the basis states of Eq. (12) are not those involved when calculating the matrix elements of the potential in Eq. (9). There, the  $\gamma$  matrices are coupled with the *quark* spin. Thus, for the quark coupling to the zero-component of the v-diquark (via  $\sim \gamma^5 \gamma^0$ ), also the following basis states with *negative* parity are needed (see App. A, B):

$$e_s^3(\hat{p}) = \begin{pmatrix} 0 \\ \chi_s \end{pmatrix}, \quad e_s^4(\hat{p}) = \begin{pmatrix} \frac{\vec{\sigma}\hat{p}}{W+M_{12}}\chi_s \\ 0 \end{pmatrix} \quad (13)$$

Then,

$$\begin{aligned} \gamma^5 \gamma^0 e_s^3(\hat{p}) &\sim e_s^1(\hat{p}) \\ \gamma^5 \gamma^0 e_s^4(\hat{p}) &\sim e_s^2(\hat{p}) \\ \gamma^5 [\gamma^{[1]} \otimes e^1(\hat{p})]_s^{\frac{1}{2}} &\sim e_s^1(\hat{p}) \\ \gamma^5 [\gamma^{[1]} \otimes e^2(\hat{p})]_s^{\frac{1}{2}} &\sim e_s^2(\hat{p}) \end{aligned} \quad (14)$$

are basis states of positive parity, see also App. A.

### III. CURRENT MATRIX ELEMENTS AND FORM FACTORS

The current matrix elements are calculated in the Mandelstam formalism [15]. The corresponding diagrams are shown in Fig. 2. For details of the calculation see [1]. The Bethe-Salpeter amplitudes of course depend on the total and relative momenta  $P$  and  $p$ . A correct boost prescription (App. B) for the outgoing  $\bar{\chi}$  is crucial for a relativistic treatment. Since the implementation of this formalism in the scalar diquark-quark model is elaborated in [1] we only focus our attention on aspects concerning the v-diquark.

The propagator of the v-diquark is chosen to be

$$\Delta_1^F(p)_{\mu\nu} = -i \frac{g_{\mu\nu}}{p^2 - m^2 + i\epsilon}, \quad (15)$$

i.e. we neglect the term  $\frac{p_\mu p_\nu / m^2}{p^2 - m^2 + i\epsilon}$ . There are several technical reasons for this: Firstly, it is not possible to state a coordinate independent normalization condition with the full propagator. Secondly, in the Salpeter approach, the contour integral over the full propagators in the complex plane does not vanish at infinity for the components with  $\mu, \nu = 0$ . Finally, the full propagator has no inverse, which, however, is mandatory in the Salpeter approach, e.g. to derive the normalization condition.

The coupling of a photon with index  $\mu$  to a massive vector particle is given by [17,18] (see Fig. 3):

$$\begin{aligned} \Gamma_{\mu; ba} = & -(p + p')_\mu g_{ba} + ((1 + \kappa)p_b - (\kappa + \xi)p'_b) g_{\mu a} \\ & + ((1 + \kappa)p'_a - (\kappa + \xi)p_a) g_{\mu b}, \end{aligned} \quad (16)$$

with  $\kappa$  the anomalous contribution to the magnetic moment of the v-diquark ( $\kappa = 1$  for a pointlike spin-1 particle).  $\xi$  is a gauge parameter introduced by Lee and Yang. According to our choice of the propagator (15) we put  $\xi = 1$ . We will see that this is indeed necessary to obtain a conserved current and the correct normalization of the Salpeter amplitudes.

Rewriting the coupling (16) in spherical components with indices  $a$  and  $b$  and taking into account the corresponding Clebsch-Gordan coefficients, we obtain the coupling matrices in the space of  $(e^{[0]}, e_0^{[1]}, e_V^{[1]})$ , i.e. including both scalar and v-diquark couplings:

$$\Gamma^0 = \begin{pmatrix} (p_1 + p'_1)^0 & 0 & 0 \\ 0 & -(p_1 + p'_1)^0 & \frac{1}{\sqrt{3}}(1 + \kappa) q \\ 0 & -\frac{1}{\sqrt{3}}(1 + \kappa) q & (p_1 + p'_1)^0 \end{pmatrix} \quad (17a)$$

$$\Gamma^3 = \begin{pmatrix} -(p_1 + p'_1)^3 & 0 & 0 \\ 0 & (p_1 + p'_1)^3 & \frac{1}{\sqrt{3}}(1 + \kappa) q^0 \\ 0 & -\frac{1}{\sqrt{3}}(1 + \kappa) q^0 & -(p_1 + p'_1)^3 \end{pmatrix} \quad (17b)$$

$$\Gamma^+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}}(1 + \kappa) q^0 \\ 0 & -\frac{1}{\sqrt{3}}(1 + \kappa) q^0 & -\frac{2}{3}(1 + \kappa) q \end{pmatrix}, \quad (17c)$$

where  $\Gamma^+ = \frac{1}{2}(\Gamma^1 + i\Gamma^2)$ .  $(q^0, \vec{q})$  is the photon momentum and  $q = |\vec{q}| = -q^3$ . Transitions between scalar and v-diquarks are neglected since they would change the flavour symmetry in our model [13,18]. From Eq. (17a) we see that, in accordance with the choice of the propagator, the zero component of the v-diquark contributes negatively to the norm. As in [1] we see that the current is conserved exactly also in the v-diquark channel.

In the non-relativistic quark-model the spin-flavour wave function of the nucleon for zero angular momentum is totally-symmetric, see App. C. We identify

$$\begin{aligned}\mathcal{R}(p) \left[ \left[ \chi^{\frac{1}{2}} \otimes \chi^{\frac{1}{2}} \right]^0 \otimes \chi^{\frac{1}{2}} \right]_s^{\frac{1}{2}} &\rightarrow \Psi^{[0]}(\vec{p}) \\ \mathcal{R}(p) \left[ \left[ \chi^{\frac{1}{2}} \otimes \chi^{\frac{1}{2}} \right]^1 \otimes \chi^{\frac{1}{2}} \right]_s^{\frac{1}{2}} &\rightarrow \Psi^{[1]}(\vec{p}).\end{aligned}\tag{18}$$

One then obtains for the current matrix elements of the proton and neutron, respectively (see App. C):

$$\begin{aligned}\langle \Psi_p | j_\mu | \Psi_p \rangle &= \frac{1}{3} \langle \Psi^{[0]} | j_\mu^{diquark} | \Psi^{[0]} \rangle + \frac{2}{3} \langle \Psi^{[0]} | j_\mu^{quark} | \Psi^{[0]} \rangle \\ &\quad + \langle \Psi^{[1]} | j_\mu^{diquark} | \Psi^{[1]} \rangle \\ \langle \Psi_n | j_\mu | \Psi_n \rangle &= \frac{1}{3} \langle \Psi^{[0]} | j_\mu^{diquark} | \Psi^{[0]} \rangle - \frac{1}{3} \langle \Psi^{[0]} | j_\mu^{quark} | \Psi^{[0]} \rangle \\ &\quad - \frac{1}{3} \langle \Psi^{[1]} | j_\mu^{diquark} | \Psi^{[1]} \rangle + \frac{1}{3} \langle \Psi^{[1]} | j_\mu^{quark} | \Psi^{[1]} \rangle.\end{aligned}\tag{19}$$

Note that in the case of the proton, the photon coupling to the quark in the v-diquark channel drops out, whereas in the case of the neutron the current is sensitive to the difference between the quark and the diquark coupling.

#### IV. RESULTS AND DISCUSSION

The values of the parameters used in our model are given in Tab.I. The quark and diquark masses are the same as in our previous work [1]. The mass of the v-diquark is chosen to be equal to the scalar diquark mass. Also, the quark-diquark coupling constants are chosen to be equal:  $g = g_s = g_v$ . Their value is fixed by the minimum of the  $M(\alpha)$  curve in the Ritz variational principle, analogous to [1]. For six radial basis functions we have a minimum at  $M(\alpha) = 939$  MeV for the oscillator parameter  $\alpha = 1.35$  fm. With equal masses and couplings we are closest to the  $SU(2)_{spin} \otimes SU(2)_{isospin}$  limit which is only broken by the different types of coupling ( $\sim 1$  for scalar and  $\sim \gamma^\mu$  for the v-diquark) (Eqs. (7) and (9)). Indeed, we find in this most symmetric case a best description of all electromagnetic form factors. The parameter influencing the shape of the form factors mostly is the diquark form factor parameter  $\lambda$ , see [1]. It is put equal for both diquark types and is chosen to give a best description of the electric form factor of the proton. We obtain  $\lambda = 0.24$  fm, in agreement with the commonly used diquark size [6]. The proton electric form

factor is shown in Fig. 4 for momentum transfers up to  $-q^2 = 3 \text{ GeV}^2$ . We find a very good agreement with the experimentally found dipole shape. However, for about  $-q^2 > 3.5 \text{ GeV}^2$ ,  $G_E^p$  becomes negative, though with small absolute value and converging to zero. This change of sign is the case for all four form factors and is model inherent. From the slope at  $q^2 = 0$  we get for the rms-radius of the proton

$$\sqrt{\langle r^2 \rangle_p} = 0.84 \text{ fm} , \quad (20)$$

compared to the experimental  $\sqrt{\langle r^2 \rangle_p} = (0.862 \pm 0.012) \text{ fm}$  [19]. In Fig. 5 the neutron electric form factor is shown for momentum transfers up to  $-q^2 = 0.75 \text{ GeV}^2$  where  $G_E^n$  is well established experimentally. We obtain a remarkable description of the experimental data. For the mean square charge radius of the neutron we find

$$\langle r^2 \rangle_n = -0.118 \text{ fm}^2 , \quad (21)$$

which fits the experimental  $\langle r^2 \rangle_n = -0.119 \pm 0.002 \text{ fm}^2$  [20]. Since the neutron current is the difference of currents (see Eq. (19)), it is very sensitive to the parameters. The dashed curve shows  $G_E^n$  for a v-diquark mass  $m_v = 670 \text{ MeV}$ . Indeed, we describe the data well in the mass-symmetric case. In Fig. 6 we compare our results (full curve) with our recent calculation only involving scalar diquarks [1] (dotted curve). The improvement is obvious.

Fig. 7 shows the magnetic form factor of the proton. As can be seen from Eq. (17c), the spin flip current of the v-diquark is proportional to  $(1 + \kappa)$ ,  $\kappa$  being the anomalous magnetic moment of the v-diquark. This is chosen to be  $\kappa = 1.6$  to fit the experimental data for  $G_M^p$ . Again, our calculation reproduces the dipole shape very nicely. The dependence of  $G_M^p$  on  $\kappa$  is shown by the dashed curves. The upper dashed curve corresponds to  $\kappa = 2.0$ , the lower one to  $\kappa = 1.0$ . The larger the diquark's anomalous magnetic moment the larger is the absolute value of the magnetic form factor. The neutron magnetic form factor is shown in Fig. 8. The calculation matches the experimental data qualitatively. However, the absolute value of the calculated curve is too small. Extrapolating to  $q^2 = 0$ , we get from  $G_M^N(0)$  for the magnetic moments of the proton and neutron:

$$\begin{aligned} \mu_p &= 2.78 \mu_N \quad (\text{exp.: } 2.793 \mu_N) \\ \mu_n &= -1.51 \mu_N \quad (\text{exp.: } -1.913 \mu_N) \\ \rightarrow \frac{\mu_n}{\mu_p} &= -0.543 \quad (\text{exp.: } -0.685) . \end{aligned} \quad (22)$$

The  $SU(6)$  quark model prediction is  $-\frac{2}{3}$ .

In Fig. 9 we show the zero-components of the various currents (see Fig. 2). As shown analytically in [1], the corresponding quark and diquark currents have to be equal at  $q^2 = 0$ , thus equally contributing to the normalization of the Salpeter amplitude (Eq. (5)). It is interesting to compare these contributions of the three channels (scalar : zero-component of v-diquark : vector-component of v-diquark):



$$0.63 : -0.23 : 0.60 , \quad (23)$$

which reproduces the result of [12] in the mass-symmetric case (where both components of the v-diquark-channel are added):  $0.64 : 0.36$ .

The four spin flip currents are shown in Fig. 10. It is mainly the current  $\langle \Psi^{[1]} | j_+^{diquark} | \Psi^{[1]} \rangle$  (lowest dashed curve) which improves our results for the magnetic form factors compared to our recent work [1]. It is interesting to see that at  $-q^2 > 1 \text{ GeV}^2$  the spin flip current, i.e. the magnetic form factor, is essentially due to the spin flip of the v-diquark. The transitions between the zero-component and the vector-component of the v-diquark, corresponding to the non-diagonal elements in the coupling matrices of the Eqs. (17a)-(17c) are seen to add up to zero:

$$\langle \Psi_0^{[1]} | j_\mu^{diquark} | \Psi_V^{[1]} \rangle = - \langle \Psi_V^{[1]} | j_\mu^{diquark} | \Psi_0^{[1]} \rangle , \quad (24)$$

see the dotted curves in Fig. 9 for  $\mu = 0$ .

## V. SUMMARY AND OUTLOOK

We developed a formally covariant constituent quark-diquark model of the nucleon. Two kinds of diquarks, scalar and axial-vector ones, are taken into account. The only interaction considered is a quark exchange. Starting from the Bethe-Salpeter equation for a bound state of a quark and a massive boson, we obtain in the instantaneous approximation for the interaction kernel a pair of coupled Salpeter-equations. As a further approximation we choose a scalar-vector symmetric form of the spin-1 propagator. The equations are solved involving the Ritz variational principle with a finite number of radial basis functions. To obtain a stable solution, a form factor of the diquark has to be introduced. The current matrix elements are calculated in the Mandelstam formalism. With five parameters in the scalar-vector symmetric case (constituent quark and diquark masses, diquark size parameter  $\lambda$ , diquark-quark coupling  $g$  and the anomalous magnetic moment of the axial-vector diquark) we find an excellent agreement with the experimental electromagnetic form factors for momentum transfers up to  $-3 \text{ (GeV/c)}^2$ . Only the magnetic form factor of the neutron is a little off the experimental curve. The calculated charge radii agree with the experimental ones, and we find the correct anomalous magnetic moment of the proton.

It is indeed surprising to find such a good correspondance with the experimental data in such a simple model. To what extent this is due to the chosen quark-exchange interaction, which is essentially  $\sim \frac{1}{q}$  ( $\sim \frac{1}{r^2}$  in coordinate space), remains to be examined. One important ingredient is the covariant treatment and the correct boosting of the outgoing amplitudes. In what sense our results may provide a revealing of a probable diquark-structure in the nucleon is unclear. In any case, by introducing an effective potential and diquark parameters, the complicated three-body problem can be reduced to an effective two-body one.

A further important test of our model is the calculation of  $N\text{-}\Delta$  transition form factors. In our model only the axial-vector diquark channel will contribute to this process. So the involved parameters should be further constrained.

**Acknowledgements:** I am grateful to H.R. Petry, B.C. Metsch and C.R. Münz for many helpful discussions and T. Schirmer for reading the manuscript. This work was supported by the Graduiertenkolleg 'Die Erforschung subnuklearer Strukturen der Materie'.

# TABLES

$m_q$	$m_s$	$m_v$	$g$	$\lambda$	$\kappa$
350 MeV/c <sup>2</sup>	650 MeV/c <sup>2</sup>	650 MeV/c <sup>2</sup>	8.10	0.24 fm	1.6

TABLE I. The parameters of the model

# FIGURES

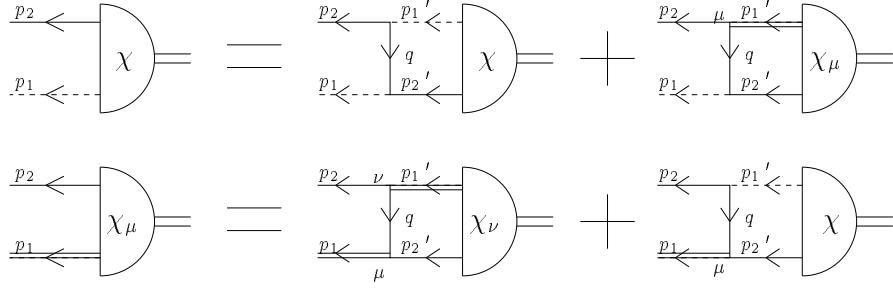


FIG. 1. The (Bethe-)Salpeter equations for a bound state of a quark and a scalar or axial-vector diquark with a quark exchange interaction, see Eq. (9). The dashed line represents a scalar diquark and the double-lined an axial-vector diquark.

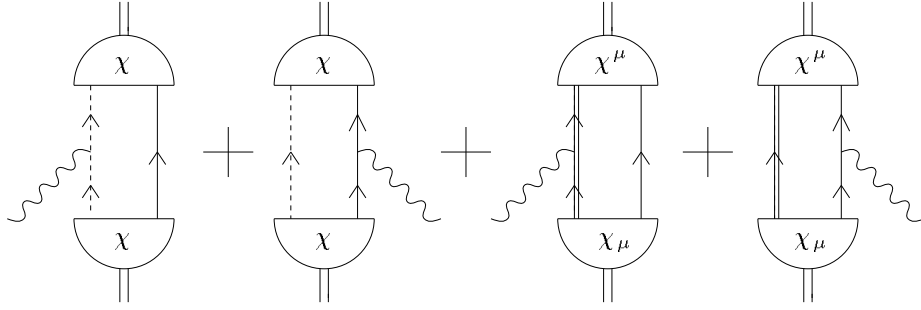


FIG. 2. The electromagnetic current is the sum of the diquark currents and the quark currents.

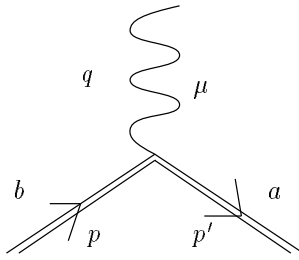


FIG. 3. The coupling of the photon to a massive vector particle (Eq. (16)).

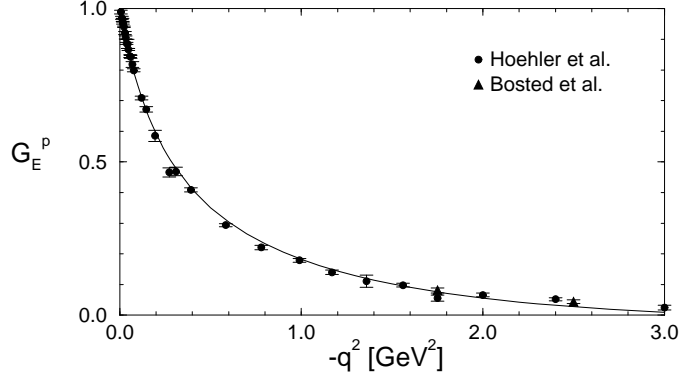


FIG. 4. The electric form factor of the proton  $G_E^p(q^2)$ ; experimental data are taken from [21,22].

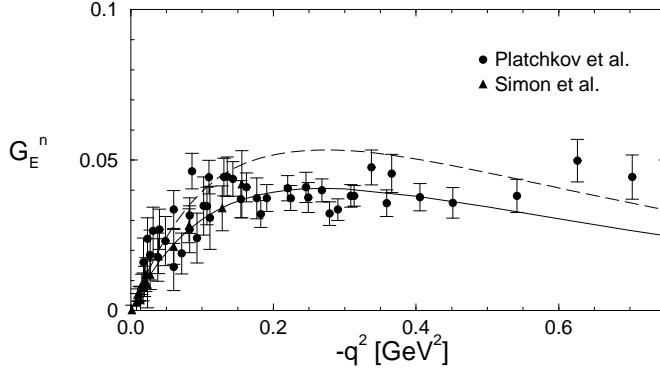


FIG. 5. The electric form factor of the neutron  $G_E^n(q^2)$ ; experimental data are taken from [19,23]. The full line corresponds to the parameter set of Tab. I, the dashed line corresponds to  $m_v = 670$  MeV.

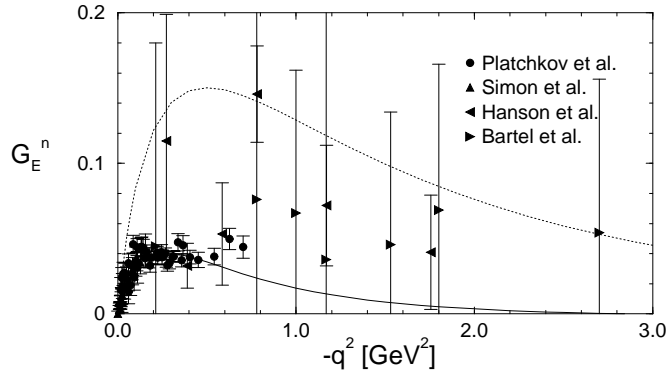


FIG. 6. The electric form factor of the neutron  $G_E^n(q^2)$ ; experimental data are taken from [19,23–25]. The full line corresponds to the parameter set of Tab. I, the dotted line is our calculation with scalar diquarks only (from [1]).

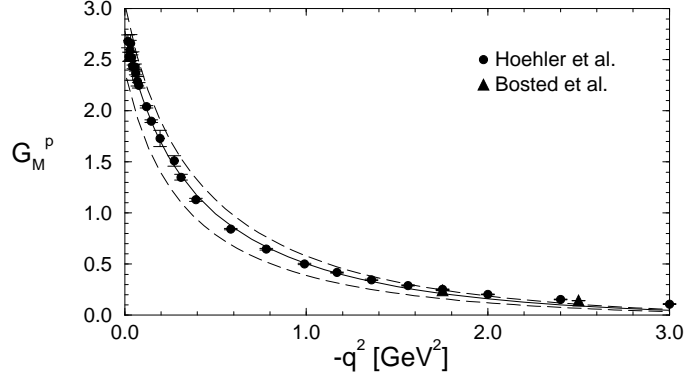


FIG. 7. The magnetic form factor of the proton  $G_M^p(q^2)$ ; experimental data are taken from [21,22]. The full line corresponds to the parameter set of Tab. I, the upper and lower dashed curve to  $\kappa = 2.0$  and  $\kappa = 1.0$ , respectively.

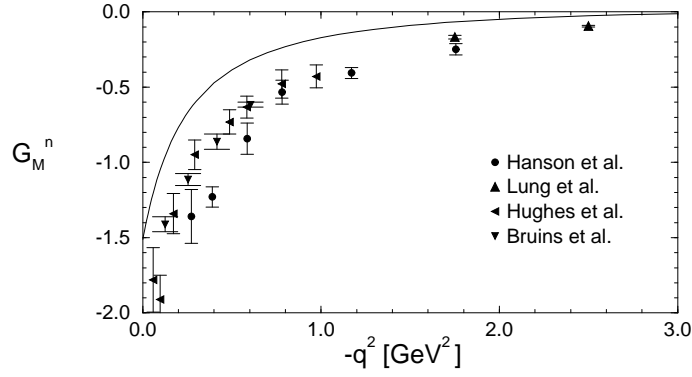


FIG. 8. The magnetic form factor  $G_M^n(q^2)$  of the neutron; experimental data are taken from [24,26–28].

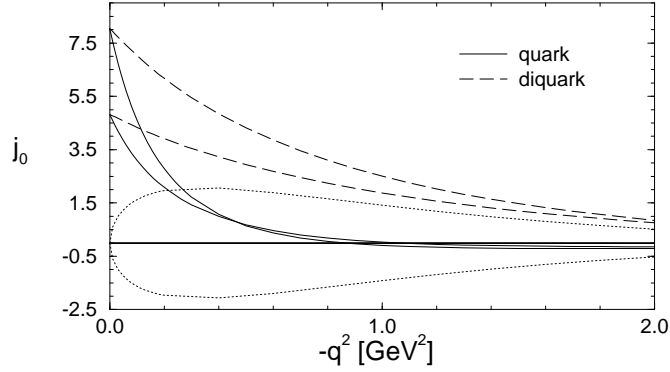


FIG. 9. The charge density  $j_0(q^2)$  of the four currents of Fig. 2 in dimensionless units. The full lines describe the coupling to the quark, the dashed lines that to the diquark. The upper pair of full and dashed lines corresponds to the currents in the scalar diquark channel, the lower one to those in the v-diquark channel (with zero- and vector components added). The dotted lines correspond to the non-diagonal couplings of Eq. (17a).

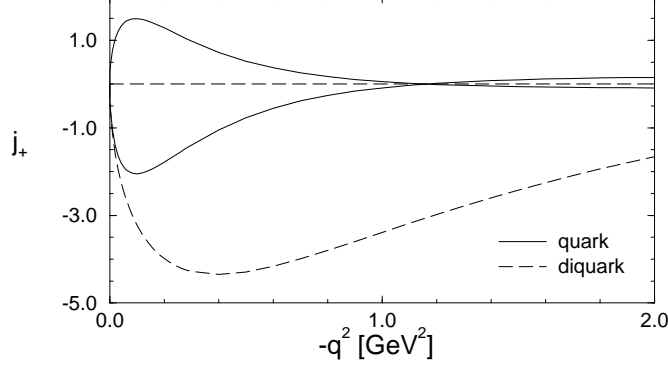


FIG. 10. The spin-flip currents  $j_+ = \frac{1}{2}(j_1 + i j_2)$  (see Fig. 2) in dimensionless units. The full lines describe the coupling to the quark, the dashed lines that to the diquark. The full line starting with negative values is the quark current in the scalar diquark channel. The  $j_+$  current of the scalar diquark is trivially zero.

## APPENDIX A: PARITY TRANSFORMATION

An axial-vector field transforms under parity-operation like [29]

$$\begin{aligned} \mathcal{P} A^\mu(x) \mathcal{P} &= -A_\mu(\tilde{x}) \\ \tilde{x}^\mu &= x_\mu . \end{aligned} \tag{A1}$$

Then, we find for the transformation of the Bethe-Salpeter amplitude:

$$\chi_P(p)_\mu = -\pi_{\mathcal{P}} \gamma^0 \chi_{\tilde{P}}(\tilde{p})^\mu . \tag{A2}$$

## APPENDIX B: LORENTZ TRANSFORMATION

For the Lorentz transformation of the v-diquark-quark Bethe-Salpeter amplitude we have [29]

$$\chi_P(p)_\mu = \Lambda^{-1}{}^\nu{}_\mu S_\Lambda^{-1} \chi_{\Lambda P}(\Lambda p)_\nu . \tag{B1}$$

However, as a spinor field, a nucleon field transforms like

$$\Psi_P(p) = S_\Lambda^{-1} \Psi_{\Lambda P}(\Lambda p) . \quad (\text{B2})$$

This correct transformation is achieved by the covariant coupling

$$\Psi_P(p) := \gamma^\mu \chi_P(p)_\mu . \quad (\text{B3})$$

### APPENDIX C: NUCLEON WAVE FUNCTION

Following Eq. (18) we write for the proton and neutron wavefunctions (without the totally anti-symmetric colour wavefunction)

$$\begin{aligned} \Phi_p(\vec{p})_s &= \mathcal{N} \left( \Psi_s^{[0]}(\vec{p}) \chi_{M_A}^{ud} + \Psi_s^{[1]}(\vec{p}) \chi_{M_S}^{ud} \right) \\ \Phi_n(\vec{p})_s &= \mathcal{N} \left( \Psi_s^{[0]}(\vec{p}) \chi_{M_A}^{udd} + \Psi_s^{[1]}(\vec{p}) \chi_{M_S}^{udd} \right) , \end{aligned} \quad (\text{C1})$$

where the flavour functions are mixed (anti-)symmetric:

$$\begin{aligned} \chi_{M_A}^{ud} &= \chi_0^{[0]} \\ \chi_{M_S}^{ud} &= -\frac{1}{\sqrt{3}}(\chi_0^{[1]} - \sqrt{2}\chi_{+1}^{[1]}) , \end{aligned} \quad (\text{C2})$$

with e.g.  $\chi_0^{[1]} = \frac{1}{\sqrt{2}}(ud + du)u$ .



## REFERENCES

- [1] V. Keiner, Z. Phys. A **354**, 87 (1996)
- [2] U. Vogl, Z. Phys. A **337**, 191 (1990)
- [3] R. T. Cahill, C. D. Roberts, J. Praschifka, Phys. Rev. D **36**, 2804 (1987)
- [4] C. Weiss, A. Buck, R. Alkofer, H. Reinhardt, Phys. Lett. B **312**, 6 (1993)
- [5] N. Ishii, W. Bentz, K. Yazaki, Phys. Lett. B **318**, 26 (1993)
- [6] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, D. B. Lichtenberg, Rev. Mod. Phys., Vol. **65**, No.4, 1199 (1993)
- [7] D. B. Leinweber, Phys. Rev. D **47**, 5096 (1993)
- [8] E. E. Salpeter, Phys. Rev. **87**, 328 (1952)
- [9] D. B. Lichtenberg, Phys. Rev. **178**, 2197 (1969)
- [10] S. Huang, J. Tjon, Phys. Rev. C **49**, 1702 (1994)
- [11] A. Buck, R. Alkofer, H. Reinhardt, Phys. Lett. B **286**, 29 (1992)
- [12] H. Meyer, Phys. Lett. B **337**, 37 (1994)
- [13] G. Hellstern, C. Weiss, Phys. Lett. B **351**, 64 (1995)
- [14] C. Hanhart, S. Krewald, Phys. Lett. B **344**, 55 (1995)
- [15] S. Mandelstam, Proc. Roy. Soc. **233**, 248 (1955)
- [16] E. E. Salpeter, H. A. Bethe, Phys. Rev. **84**, 132 (1951)
- [17] T. D. Lee, C. N. Yang, Phys. Rev. **128**, 885 (1962)
- [18] P. Kroll, M. Schürmann, W. Schweiger, Z. Phys. A **338**, 339 (1991)
- [19] G. G. Simon, C. Schmitt, V. H. Walther, Nucl. Phys. **A364**, 285 (1981)
- [20] P. E. Bosted, Phys. Rev. C **51** 409, (1995)
- [21] G. Höhler *et al.*, Nucl. Phys. **B114**, 505 (1976)
- [22] P. E. Bosted *et al.*, Phys. Rev. Lett. **68**, 3841 (1992)
- [23] S. Platchkov *et al.*, Nucl. Phys. **A510**, 740 (1990)
- [24] K. M. Hanson *et al.*, Phys. Rev. D **8**, 753 (1973)
- [25] W. Bartel *et al.*, Nucl. Phys. **B58**, 429 (1973)
- [26] A. Lung *et al.*, Phys. Rev. Lett. **70**, 718 (1993)
- [27] E. B. Hughes *et al.*, Phys. Rev. B **139**, 458 (1965)
- [28] E. E. W. Bruins *et al.*, Phys. Rev. Lett **75**, 21 (1995)
- [29] C. Itzykson, J.-B. Zuber, Quantum Field Theory, McGraw-Hill, New York (1980)